A Method for Computing Infiltration and Redistribution in a Discretized Moisture Content Domain

US Army Corps of Engineers
Engineer Research and Development Center

Cary Talbot ~ Coastal & Hydraulics Laboratory, US Army Engineer Research & Development Center, Vicksburg, MS ~ Cary.Talbot@us.army.mil
Fred Ogden ~ Department of Civil & Architectural Engineering, The University of Wyoming, Laramie, WY ~ fogden@uwyo.edu

Overview

A new discretized water content infiltration and redistribution method is proposed as a robust and computationally efficient alternative to the Richards' equation (RE) for infiltration simulation.

- Soil water content domain is discretized into hypothetical hydraulically-interacting bins
- Explicit infiltration and drainage approximations based on capillary and gravitational driving forces simulate the entry and propagation of displacement fronts
- Wetting front advances within bins create water deficits that are satisfied by capillary-driven inter-bin flow
- Numerical stability inherent to method by precluding need to directly estimate non-linear gradients
- We compare the method to RE solutions of theoretical, laboratory and field data in well-drained and high water table conditions
- New method can reproduce RE and provide a large reduction in computational effort with unconditional conservation of mass

Introduction

Infiltration models often use *Richards'* [1931] equation (RE) to simulate fluxes in the vadoze zone. RE in 1D mixed form is:

θ is moisture content [L^3L^{-3}], t is time [T], Z is depth [L], $K(\theta)$ is unsaturated hydraulic conductivity [LT^{-1}], and ψ is capillary pressure [L]

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial Z}\left(K(\theta)\left(\frac{\partial\psi}{\partial Z}+1\right)\right) \qquad (1)$$

Challenges

Using RE to simulate flux of water in vadose zone poses many challenges to infiltration models due to:

- Highly nonlinear relationships between θ, $K(\theta)$ and ψ impose significant computational burden to numerical solvers, even with current techniques and resources [*Ross*, 1990; *Smith et al.*, 1993; *Corrandini et al.*, 1997; *Miller et al.*, 1998; *van Dam and Feddes*, 2000; *Farthing et al.*, 2003; *Bashir et al.*, 2007]
- Simplifying numerical techniques can have restrictive assumptive conditions or are complex in application
- Parameters in RE are point-specific yet it is often applied to larger scales [*Beven*, 2001]

Our Approach

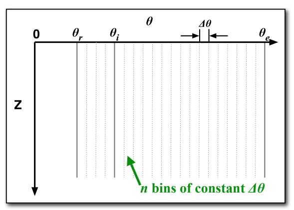
Develop a straight-forward and simple method of simulating infiltration and redistribution that:

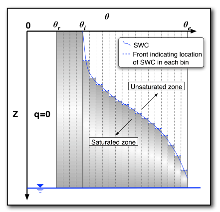
- Is computationally efficient
- Requires no additional parameterization than is used for RE
- Can accurately simulate vadose zone flux for variety of boundary and initial conditions including high and rising water tables
- Produces results similar in accuracy and detail to RE
- Can be applied to the areally-averaged infiltration case

A Discretized Moisture Content Domain

We discretize in moisture content (θ-space) instead of in elevation (Z) using n "bins" of constant water content width, $\Delta\theta$

- n bins between residual moisture content, θ_r, and porosity, θ_e
- Bins in intimate contact allowing inter-bin flow at any depth as dictated by capillarity
- Similar in concept to *Steenhuis et al.* [1990] who chose bins based on velocity to model solute transport





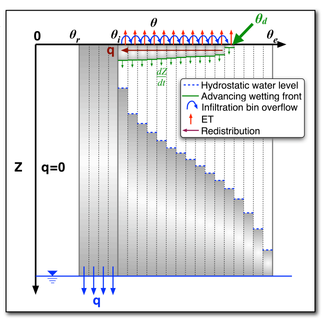
Movement of Fronts

At any given depth each bin is either fully saturated or unsaturated. The Z-value at which the saturation state changes represents either a wetting or drying front. Bin fronts can move by:

- influence of gravity and capillarity
- interaction with boundary conditions
- interaction with other bins through redistribution

Movement of fronts in a bin k is calculated the following (based on equation (26) from *Smith et al.* [1993]):

$$\frac{dZ_k}{dt}=\frac{1}{(\theta_e-\theta_i)}\left(\frac{K(\theta_d)\psi(\theta_d)}{Z_k}+K(\theta_d)\right) \qquad (2)$$

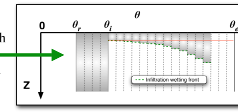


- Flux boundary conditions either deliver water to or remove water from bins at land surface
- Root distribution function can simulate ET loss from bins at any specified depth
- During precipitation periods, water is infiltrated into bins in left-to-right fashion
- Water to satisfy the demand created by an advancing front is supplied from the total volume to be infiltrated during precipitation or from water in bins to right during rainfall hiatus
- θ_d is right-most bin containing water thus $K(\theta_d)$ and $\psi(\theta_d)$ in (2) are constant for all bins in a given dt
- Using $K(\theta_d)$ assumes water will always travel downward by gravity through the largest-pore bins possible
- Using $\psi(\theta_d)$ assumes redistribution will always satisfy the soil suction of all pores in bins to the left of θ_d leaving only $\psi(\theta_d)$ unsatisfied and is thus the suction felt by the entire wetting front
- $\psi(\theta)$ values computed from *Brooks and Corey* [1966] (BC) or *van Genuchten* [1980] (vG) parametric soil water characteristic (SWC) models
- At θ values near saturation, $\psi(\theta)$ values in (2) are replaced by G_{eff}, the effective capillary drive as defined by *Morel-Seytoux et al.* [1996, equations (13) and (15)] when $G_{eff} > \psi(\theta_d)$

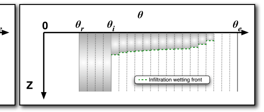
Redistribution

- $K(\theta)$ values in right-side bins increase more rapidly than $\psi(\theta)$ values decrease so water preferentially infiltrates through bins to the right as shown here
- Method assumes capillary demand from left-side bins acts immediately on water found at depth in bins to the right and is satisfied with water from right-most bin
- Deeper wetting fronts in any bin to right of left-most unsaturated bin (as denoted by red line above) are proportionately redistributed to all unsaturated bins to the left based on bin $\psi(\theta)$ values
- Redistribution is repeated in recursive fashion until no inter-bin flow occurs

Infiltration profile prior to redistribution



Infiltration profile after redistribution



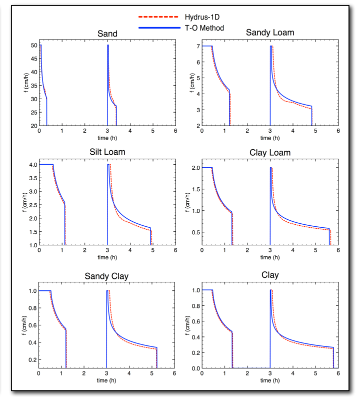
Evaluation of Method

We evaluated our method against Hydrus-1D [*Simunek et al.*, 2005] RE solutions on theoretical, laboratory, and field infiltration data sets in both well-drained and high water table conditions. We also tested for bin and temporal resolution convergence.

Comparison with RE in Well-drained Soils

- Two-pulse, 6 hour simulation of precipitation intensity sufficient to cause ponding with hiatus to allow redistribution and ponded water to infiltrate
- Second pulse begins at t=3 h with same intensity as first pulse
- Soil parameters for the 11 tested soil textures from *Rawls et al.* [1982, 1983]

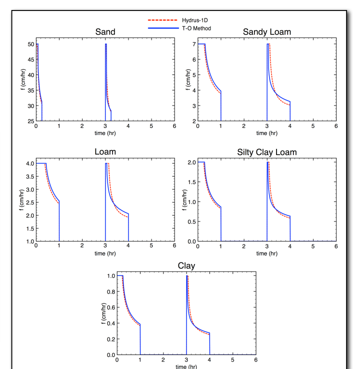
Soil	Rain Pulse	Rainfall Intensity (cm/hr)	Richards' Equation $t_{p,RE}$ (h)	F_{RE} (cm)	Talbot and Ogden $t_{p,T\text{-}O}$ (h)	$F_{T\text{-}O}$ (cm)	Δt_p (h)	ε_F (%)	ε_{RMS} (%)
Sand	1	50	0.063	63.70	0.082	70.15	-0.019	-10.13	1.22
	2	50	3.036	114.10	3.044	139.10	-0.008	-21.91	4.42
	Overall								3.24
Loamy Sand	1	15	0.177	38.40	0.191	39.15	-0.014	-1.94	2.74
	2	15	3.081	66.80	3.047	75.08	0.035	-12.40	6.74
	Overall								5.15
Sandy Loam	1	7	0.411	31.50	0.456	33.22	-0.044	-5.47	6.13
	2	7	3.114	53.63	3.063	60.82	0.051	-13.40	5.55
	Overall								5.85
Loam	1	4	0.625	18.75	0.698	18.82	-0.073	-0.38	6.36
	2	4	3.165	32.00	3.098	35.13	0.067	-9.79	7.59
	Overall								7.00
Silt Loam	1	4	0.563	15.40	0.610	16.35	-0.048	-6.14	5.38
	2	4	3.131	27.30	3.063	29.25	0.069	-7.13	6.52
	Overall								5.98
Sandy Clay Loam	1	2	0.642	17.23	0.708	17.08	-0.067	1.43	5.34
	2	2	3.171	27.48	3.100	29.83	0.071	-8.56	6.59
	Overall								6.00
Clay Loam	1	2	0.456	15.30	0.419	14.78	0.038	3.38	5.23
	2	2	3.085	23.40	3.044	24.14	0.042	-3.17	6.76
	Overall								6.04
Silty Clay Loam	1	2	0.639	14.10	0.679	13.17	-0.040	6.62	3.31
	2	2	3.164	22.50	3.081	22.99	0.083	-2.19	4.76
	Overall								4.10
Sandy Clay	1	1	0.488	22.23	0.540	19.18	-0.053	13.68	4.63
	2	1	3.129	34.13	3.051	31.31	0.078	8.26	6.12
	Overall								5.43
Silty Clay	1	2	0.258	15.40	0.268	14.37	-0.010	6.70	2.17
	2	2	3.028	22.05	3.007	21.42	0.021	2.85	6.60
	Overall								4.91
Clay	1	1	0.429	15.00	0.469	12.79	-0.040	14.73	4.27
	2	1	3.092	21.50	3.028	20.13	0.064	6.37	6.19
	Overall								5.32
			Average Differences		First Pulse		-0.034	2.04	4.25
					Second Pulse		0.052	-5.55	6.17
					Overall				5.36



Comparison with RE in High Water Table Soils

- Same two-pulse simulations as above tested for five of the soils spanning textural and performance range but with an initial condition of a water table in hydrostatic equilibrium at depth of 150 cm
- There is generally better agreement between the Talbot-Ogden method and RE in high water table conditions than in the well-drained conditions

Soil	Rain Pulse	Rainfall Intensity (cm/hr)	Richards' Equation $t_{p,RE}$ (h)	F_{RE} (cm)	Talbot and Ogden $t_{p,T\text{-}O}$ (h)	$F_{T\text{-}O}$ (cm)	Δt_p (h)	ε_F (%)	ε_{RMS} (%)
Sand	1	50	0.056	63.90	0.081	68.47	-0.025	-7.16	1.11
	2	50	3.035	117.90	3.050	125.15	-0.015	-6.15	1.86
	Overall								1.53
Sandy Loam	1	7	0.271	45.75	0.315	45.09	-0.044	1.45	1.95
	2	7	3.115	71.55	3.058	79.27	0.057	-10.79	5.36
	Overall								4.03
Loam	1	4	0.375	32.85	0.435	30.33	-0.060	7.66	2.00
	2	4	3.146	48.60	3.063	52.77	0.083	-8.58	4.46
	Overall								3.45
Silty Clay Loam	1	2	0.253	35.78	0.293	26.67	-0.040	25.45	2.04
	2	2	3.083	48.25	3.033	42.06	0.050	12.83	5.30
	Overall								4.02
Clay	1	1	0.219	20.25	0.256	18.70	-0.036	7.67	2.07
	2	1	3.067	30.35	3.026	28.50	0.040	6.10	5.02
	Overall								3.84
			Average Differences		First Pulse		-0.041	7.02	1.83
					Second Pulse		0.043	-1.32	4.40
					Overall				3.38

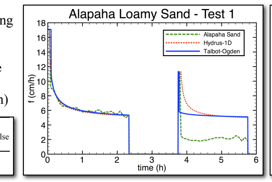


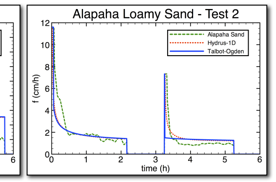
Evaluation of Method (continued)

Comparison with RE and Field Data on Well-drained Loamy Sand

- *Rawls et al.* [1976] performed two two-pulse field tests with no ponding on loamy sand using a Perdue sprinkling infiltrometer
- Talbot-Ogden and RE results agree well with each other and field data (Test 1, pulse 2 being the exception)

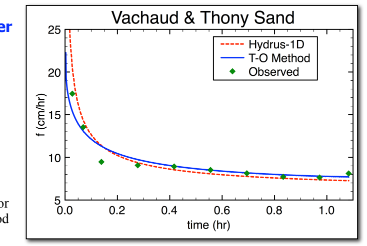
Test Number	$\varepsilon_{RMS,RE}$ First Pulse (%)	$\varepsilon_{RMS,T\text{-}O}$ First Pulse (%)	$\varepsilon_{RMS,RE}$ Second Pulse (%)	$\varepsilon_{RMS,T\text{-}O}$ Second Pulse (%)
1	6.14	7.95	33.14	27.82
2	15.70	14.35	13.33	18.81





Comparison with RE and Field Data on High Water Table Laboratory Sand

- *Vachaud and Thony* [1971] measured capillary head and soil moisture content in a laboratory sand column in hydrostatic equilibrium with a water table at a depth of 101 cm
- Constant head boundary condition of -12 cm was applied at column surface
- Infiltration rate data estimated from *Vachaud and Thony* [1971] results via area-under-the-curve calculations
- Talbot-Ogden and Hydrus-1D methods run with same BC soil parameters as calibrated by *Salvucci and Entekhabi* [1995]
- Root-mean square (RMS) infiltration flux error of 0.92 cm/hr for Talbot-Ogden method and 1.22 cm/hr for Hydrus-1D RE method



Bin and Temporal Resolution Convergence Tests

- Time step size and number of bins can affect accuracy of simulation
- Tests to determine adequate number of bins and time step size performed on hypothetical soils
- Results vary by soil texture with more bins and smaller time steps needed by coarser soils
- Most soils can be simulated with 200 bins and a time step size of 5 seconds

Textural Classification	No. of Bins, n	No. of Unsat. Bins	Time Step Size (s)	Δt_p (%)	ΔF (%)
Sand	400	387	2.5	0.00	0.19
Loamy Sand	350	331	2.5	0.24	0.22
Sandy Loam	300	256	5.0	0.00	0.04
Loam	200	156	7.5	0.49	0.34
Silt Loam	200	150	7.5	0.00	0.04
Sandy Clay Loam	200	139	7.5	0.16	0.17
Clay Loam	200	123	5.0	0.20	0.20
Silty Clay Loam	200	114	5.0	0.18	0.34
Sandy Clay	200	77	5.0	0.26	0.29
Silty Clay	200	94	5.0	0.16	0.29
Clay	200	77	5.0	0.62	0.40

Conclusions

The discretized moisture content infiltration method allows for reliable prediction of vadose zone fluxes for a variety of boundary and initial conditions in homogeneous soil conditions. It is robust, suitable and more computationally efficient than RE methods in simulating 1D infiltration problems.

- Numerical efficiency achieved by discretizing the moisture content domain, eliminating burden of numerically estimating the highly non-linear $\partial\psi/\partial Z$ and $\partial\theta/\partial\psi$ gradients
- Computationally-expensive operations such as exponentiation only required at model initiation, all others are arithmetic
- By contrast, RE solutions often require up to 16 floating point operations per iteration, per computation node
- Evaluation of method indicates it is capable of reliably matching RE solutions and observed data on hypothetical, laboratory and field soils

Future development of method will include heterogeneous soil conditions (layered soils), coarse grain and macro-pore infiltration, and areally-averaged infiltration.

Acknowledgments

This method was developed with funding support from the Coupled Surface Water-Groundwater Interaction work unit of the US Army Corps of Engineers' (USACE) System-Wide Water Resources Program (SWWRP). Permission was granted by the Chief of Engineers, USACE to present this information.

References

Bashir, R., D. F. Stolle, and J. E. Smith (2007), Flux-based alternative formulation for variably saturated subsurface flow, *J. Hydrol. Eng.*, *12*(5), 501-512.
Beven, K. (2001), How far can we go in distributed hydrological modelling?, *Hydrol. Earth System Sci.*, *5*(1), 1-12.
Brooks, R. H., and A.T. Corey (1966), Properties of porous media affecting fluid flow, *J. Irrig. Drain. Div. Am. Soc. Civ. Eng.*, *92*(IR2), 61-88.
Corradini, C., F. Melone, and R. E. Smith (1997), A unified model of infiltration and redistribution during complex rainfall patters, *J. Hydrol.*, *192*, 104-124.
Farthing, M. W., C. E. Kees, C. T. Miller (2003), Mixed finite element methods and higher order temporal approximations for variably saturated groundwater flow, *Adv. Water Resour.*, *26*(4), 373-394.
Kirkland, M. R., R. G. Hills, P. J. Wierenga (1992), Algorithms for solving Richards' equation for variably saturated soils, *Water Resour. Res.*, *28*(8), 2049-2058.
Miller, C. T., G. A. Williams, C. T. Kelley, and M. D. Tocci (1998), Robust solution of Richards' equation for non-uniform porous media, *Water Resour. Res.*, *34*(10), 2599-2610.
Morel-Seytoux, H. J., P. D. Meyer, M. Nachabe, J. Touma, M. Th. van Genuchten, and R. J. Lenhard (1996), Parameter equivalence for the Brooks-Corey and van Genuchten soil characteristics: Preserving the effective capillary drive, *Water Resour. Res.*, *32*(5), 1251-1258.
Rawls, W. J., D. L. Brakensiek, and N. Miller (1983), Green-Ampt infiltration parameters from soils data, *J. Hydr. Eng.*, *109*(1), 62-70.
Rawls, W. J., D. L. Brakensiek, and K. E. Saxton (1982), Estimation of soil water properties, *Trans. ASAE.*, *25*(5), 1316-1330.
Rawls, W. J., P. Yates, and L. Asmussen (1976), *Calibration of selected infiltration equations for the Georgia Coastal Plain*, Agricultural Research Service, ARS-S-113, U. S. Department of Agriculture, July.
Richards, L. A. (1931), Capillary conduction of liquids in porous mediums, *Physics*, *1*, 318-333.
Ross, P. J. (1990), Efficient numerical methods for infiltration using Richards' equation, *Water Resour. Res.*, *26*, 279-290.
Salvucci, G. D., and D. Entekhabi (1995), Ponded infiltration into soils bounded by a water table, *Water Resour. Res.*, *31*(11), 2751-2759.
Simunek, J., M. Th. van Genuchten, and M. Sejna (2005), The HYDRUS-1D software package for simulating the movement of water, heat, and multiple solutes and variable saturated media. Version 3.0, *HYDRUS Software Series I,* Department of Environmental Sciences, University of California Riverside, Riverside, CA.
Smith, R. E., C. Corradini, and F. Melone (1993), Modeling infiltration for multistorm runoff events, *Water Resour. Res.*, *29*(1), 133-144.
Steenhuis, T. S., J.-Y. Parlange and M. S. Andreini (1990), A numerical model for preferential solute movement in structured soils, *Geoderma*, *46*, 193-208.
Vachaud, G., and J.-L. Thony (1971), Hysteresis during infiltration and redistribution in a soil column at different initial water contents, *Water Resour. Res.*, *7*(1), 111-127.
van Dam, J. C., and R. A. Feddes (2000), Numerical simulation of infiltration, evaporation, and shallow groundwater levels with the Richards' equation, *J. Hydrol.*, *233*, 72-85.
van Genuchten, M. Th. (1980), A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am. J.*, *44*, 892-898.