A Moisture Content-Discretized Infiltration Method

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Abstract In an effort to improve the computational efficiency and robustness of one-dimensional vadose zone flow calculations, alternatives to the Richards' Equation (RE) are sought as part of on-going System-Wide Water Resources Program research by the US Army Engineer Research & Development Center (ERDC) in the development of coupled surface and subsurface flow interaction codes. This paper will describe a depth-continuous, moisture content-discretized infiltration method under development at ERDC. The range of moisture content within a given soil is discretized into vertically continuous, interactive bins. The entry and vertical movement of wetting fronts in each bin are simulated by means of explicit infiltration approximations based on capillary and gravitational driving forces. The advancement of wetting fronts within a bin create pore-water deficits that are satisfied by capillary-driven inter-bin flow. The method inherently provides numerical stability and robustness because it precludes the need to include the explicitly non-linear constitutive models estimated by other vadose zone flow codes. Comparison of method results with a RE solution will be presented.
Key words infiltration; Richards' equation; vadose zone; numerical simulation

INTRODUCTION

The US Army Corps of Engineers (USACE) and other federal and state agencies in the United States are currently engaged in the application of coupled surface water-groundwater interaction simulation models in large-scale wetland restoration projects in South Florida, coastal Louisiana and other locations. Simulations of many hundreds of square miles over multiple decades are required to evaluate long-term restoration alternatives. Modelling tools currently being employed in these projects require excessive run times due, in part, to the lack of a numerically efficient means of simulating vadose zone flow. A research effort within the USACE's System-Wide Water Resources Program (SWWRP) is funding the development of innovative approaches to infiltration modelling to address this problem.

While the processes of infiltration and redistribution of water in unsaturated soil have been studied for a century or more, a simple, concise method of simulating these processes with reliable accuracy in conditions commonly encountered in the field has remained elusive. Physically rigorous relationships such as Richards' Equation (RE) (Richards, 1911) have been developed to describe the complex relationship between moisture content and suction potential. However, these relationships require solution via computationally expensive numerical techniques (Ross, 1990; Smith *et al.*, 1993; Short *et al.*, 1995; Corrandini *et al.*, 1997). Simpler approximations and parametric forms of these equations are capable of providing similar solutions but are generally hindered by restrictive assumptions and limited applicability or require fitting of non-physical parameters to individual data sets.

In this paper we introduce a method of simulating infiltration and redistribution that is computationally efficient, requiring no more parameterization than what is commonly known or assumed for vadose zone models, and which is capable of producing results similar in accuracy and detail to equations such as RE. The method's constitutive relationships also enable it to be applied to a broad range of problems, such as gravity-dominated and macro-pore flow, for which RE and other capillary-driven infiltration equations are ill-suited to simulate.

While this method is developed primarily as a means of improving the performance of surface water-groundwater interaction studies of both large and small scales, the method is general and has potential for wide application in any model that simulates the flow of water in the vadose zone including infiltration models, hydrologic runoff models that include infiltration processes and variably-saturated groundwater flow models. This paper will describe the method in detail and provide comparisons of results and computational efficiency measures with RE solutions.

BACKGROUND

The problem of simulating the processes of infiltration and redistribution of water in the vadose zone of soils has been extensively treated in the literature with a wide range of exact and approximate solutions proposed. Here we will first examine models that provide approximate infiltration solutions, followed by exact methods.

Approximate Infiltration Models

Dozens of models have been developed based on approximate solutions which Mishra *et al.* (1999) classify into three categories: (1) physically-based models, (2) semi-empirical and (3) empirical. Physically-based models are formulated from Darcy's law and the law of conservation of mass with varying degrees of consideration given to dimensionality, soil water characteristic relationships, and initial and boundary conditions (Mishra *et al.*, 2003). Examples of such models include the models of Green and Ampt (1911), Philip (1957a, 1969), and Smith and Parlange (1978) among many others. Semi-empirical models are based on the continuity equation and simple infiltration rate-cumulative infiltration relationships, which represent a compromise between the physically-based and empirical models. Some examples include the models of Horton (1938), Holtan (1961), and Singh-Yu (1990). Empirical models are based on curve-fitting of parametric models to laboratory or field data. Some prominent examples include the Kostiakov (1932), SCS-CN, modified Kostiakov (Smith, 1972), and Collis-George (1977) models.

Despite the attention the problem of infiltration has received and the large number of approximate solution models available, the suitability of these models to real-world data is less than clear, leaving unanswered the question of which model is better and under what conditions (Mishra *et al.*, 2003). Bouwer (1969) indicates that there is often a trade-off between exactness of a solution and the restrictions on its application due to the assumptions made concerning, for example, the initial soil moisture profile.

He argues that the demonstrably inexact Green and Ampt model is in some ways more useful than other approaches because of the simplicity with which it can be applied to conditions of variability in the initial soil moisture profile and the soil texture profile (Salvucci and Entekhabi, 1995).

Exact Infiltration Simulation: The Richards' Equation

The restrictive assumptions and conditions of applicability that accompany the majority of approximate infiltration models contribute to the fact that process-based models of flow in porous media are generally based on the Richards' equation (Simunek *et al.*, 2003). The RE is given in one-dimensional, mixed-form by

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left(K(\theta)\left(\frac{\partial \psi}{\partial Z}+1\right)\right) \tag{1}$$

where θ is volumetric soil moisture content [L^3L^{-3}], $K(\theta)$ is hydraulic conductivity as a function of θ [LT^{-1}], ψ is the soil water matric potential [L], Z is depth [L], and t is time [T].

RE remains the most general method to compute soil moistures and hydrological fluxes such as infiltration, runoff (be it infiltration or saturation excess), evapotranspiration (ET) and groundwater recharge. Soil layering, shallow groundwater table and the effects of soil moisture on infiltration, often the very conditions that exclude the use of approximate methods, are all easily incorporated into the RE model solution (Downer and Ogden, 2004).

However, applying RE in hydrological studies, particularly coupled surface water-groundwater interaction models, has significant drawbacks. The RE can be computationally expensive when used to describe sharp wetting fronts during infiltration (Ross, 1990; Pan and Wierenga, 1995) and has been considered too computationally expensive for use in the context of general surface water hydrology (Ross, 1990; Smith *et al.*, 1993; Short *et al.*, 1995; Corrandini *et al.*, 1997). While recent advances in computing platform speed and the availability of parallel processing platforms has alleviated some of the computational expense, RE still presents a formidable challenge due to the spatial discretization required to achieve numerical stability.

The challenge arises from the highly non-linear nature of the unsaturated hydraulic conductivity function, $K(\theta)$. $K(\theta)$ naturally varies over a dozen or more orders of magnitude for commonly encountered soils (Leij, 1999). Directly solving RE under conditions where $K(\theta)$ is so highly non-linear is a complex challenge that requires a large numerical effort.

Another challenge presented by the large natural variation in $K(\theta)$ is manifest during dry conditions, when $K(\theta)$ is several orders of magnitude less than the saturated value. How rapidly $K(\theta)$ can increase is dependant on how rapidly soil moisture increases. If RE is being solved over a numerical grid with a relatively large spatial resolution, the small amount of water that can enter the computational cell, owing to the low value of $K(\theta)$, will have little effect on raising the soil moisture of the cell. In this case, $K(\theta)$ remains low, and little or no infiltration occurs before runoff and ET deplete the available water in that cell, leading to flux errors in the infiltration calculations.

Downer and Ogden (2004) performed a spatial grid convergence study on watersheds of varying runoff mechanisms using a 1-dimensional RE. They found that in all cases, a very fine resolution, on the order of 1.0 cm, is required near the soil surface in order to keep large numerical errors from significantly impacting the estimation of surface fluxes and soil parameter values. Their study indicated that this level of fine resolution is not needed deep into the soil column but that large cell sizes near the surface prevent the proper simulation of the soil column. This finding has serious implications for large-scale coupled surface water-groundwater interaction studies that employ RE in determining hydrological fluxes through the vadose zone.

Unstable Flow

Finally, we briefly consider the impacts of the phenomena of preferential flow on the accuracy of infiltration and redistribution simulation. Much field and laboratory observations have been made on the subject over the past several decades (Egorov *et al.*, 2003) with the conclusion that unstable flows, caused by preferential flow pathways wherein gravity-driven forces are dominant over capillary forces, can occur over the wide range of conditions that are found normally in the field. Egorov *et al.* (2003) also state that while the conditions that initiate unstable flows are not well understood, it has been shown through both linear and non-linear stability analyses of homo- and heterogeneous soils that RE is unconditionally stable and therefore incapable of simulating gravity-driven flow. In domains where soils that exhibit gravity-dominant flow are prevalent, neither RE or any of the approximate methods discussed so far would be capable of providing accurate infiltration and redistribution estimates. In their comparison of 14 different infiltration models on a large number of different soil types, including some sandy soils from Georgia (USA), Mishra *et al.* (2003), concluded that the tested models were all incapable of simulating the erratic infiltration decay pattern exhibited by these soils due to their gravity-driven nature.

A NEW SIMULATION APPROACH

Given the limitations and restrictions of using approximate methods to simulate infiltration and redistribution behaviour under real world conditions, the challenges posed by the highly non-linear RE under those same conditions and the added challenge of simulating gravity-driven, unstable infiltration in some soils, a balanced solution is sought. Such a solution would provide the robustness and wide applicability of RE for varied initial and boundary conditions, heterogeneous, and layered and uniform soils, the simplicity and computational efficiency of approximate solutions that do not solve highly non-linear equations nor require fine spatial resolution, and the ability to simulate unstable, gravity-driven infiltration and redistribution.

Discretized Moisture Content Infiltration Simulation Method

A new and innovative approach is proposed that seeks to fulfil the requirements of the described balanced solution for one-dimensional infiltration and redistribution simulation. The approach is based on a discretization of the horizontal axis, representing volumetric soil moisture content, in θ-Z space (Fig. 1). This discretization extends in Z from the land surface to the water table (for simplicity in explanation, we'll only concern ourselves with a single-layer system), and is of constant width, $\Delta\theta$ between θ_r, the residual moisture content of the medium, and θ_e, the medium's porosity. At any given depth, Z, within each $\Delta\theta$ segment, or "bin", only one of two conditions exists: either the bin is fully saturated or it is fully unsaturated. A depth at which the saturation state changes from one value to the other is marked with a "front" that represents either a wetting or drying front. Fronts can move under the influence of gravity and capillarity as well as through interaction with boundary conditions at the surface above (infiltration and ET), the water table below (rise and fall of the water table) and with each other.

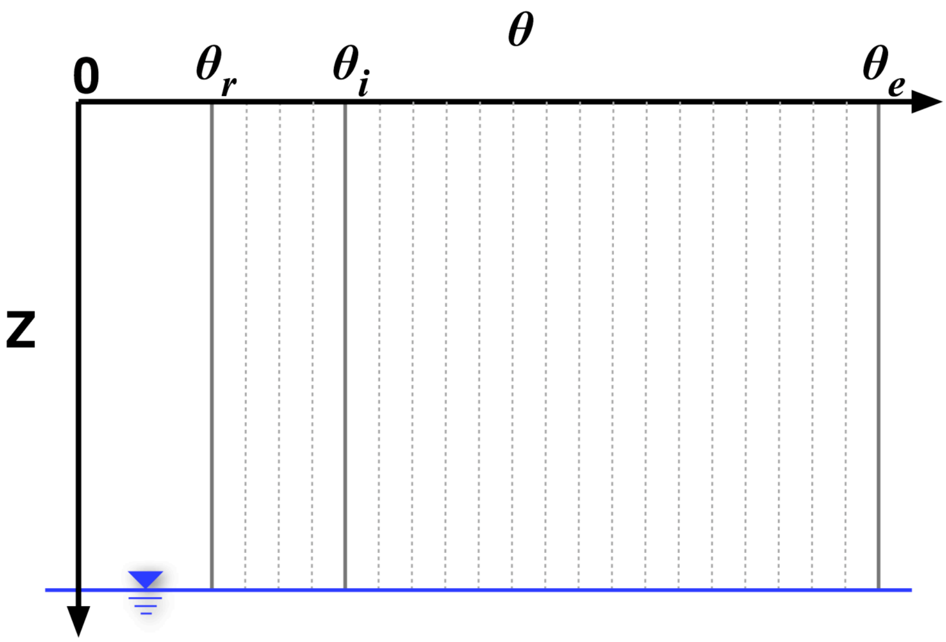

Fig. 1 Discretization of θ-Z-space into bins.

Movement of any given front in a bin is defined by the following relationship (after Ogden and Saghafian, 1997):

$$\frac{dZ_i}{dt} = \frac{2}{(\theta_e - \theta_i)}\left(\frac{Ks_i G_d}{Z_i} + Ks_i\right) \tag{2}$$

where dZ_i/dt is the i-th bin front speed in the Z-direction [LT^{-1}], Ks_i is the bin saturated hydraulic conductivity [LT^{-1}], θ_e is the porosity [L^3L^{-3}], θ_i is the initial soil moisture content [L^3L^{-3}], G_d is the soil matric (suction) potential of the smallest-pore bin that is not yet saturated [L], and Z_i is the length of the saturated portion of the bin between the current wetting and drying front [L].

The bin-specific saturated hydraulic conductivity, Ks_i is defined by Jackson's (1972) formulation of the Childs and Collis-George (1950) equation for predicting unsaturated hydraulic conductivity. This is given by:

$$K(\theta_j) = K_s\left(\frac{\theta_j + \frac{\Delta\theta}{2}}{\theta_s}\right)\frac{\sum_{j=i}^{M}\frac{2j+1-2i}{\psi_j^2}}{\sum_{j=1}^{M}\frac{2j-1}{\psi_j^2}} \tag{3}$$

where K_s is the saturated hydraulic conductivity of the entire medium [LT^{-1}], $\theta_j = (\theta_s - j*\Delta\theta)$, θ_s is the mid-bin θ value, M is the number of bins and ψ is the suction head [L] corresponding to θ_s, as defined by either a van Genuchten (1980) or Brooks and Corey

(1964) parametric soil water characteristic (SWC) curve.

The soil matric potential for each bin, G_i, is computed by integrating between the ψ -values obtained from the chosen SWC using the left- and right-edge θ-values for each bin.

Initially, the bins between θ_r and θ_i, the initial condition moisture content, are fully saturated from the land surface to the water table. The remaining bins can be specified with any initial soil moisture condition desired. A uniform, dry vadose zone is characterized by empty bins between θ_i and θ_e. A hydrostatic condition is represented by a set of fronts such as those shown in Fig. 2.

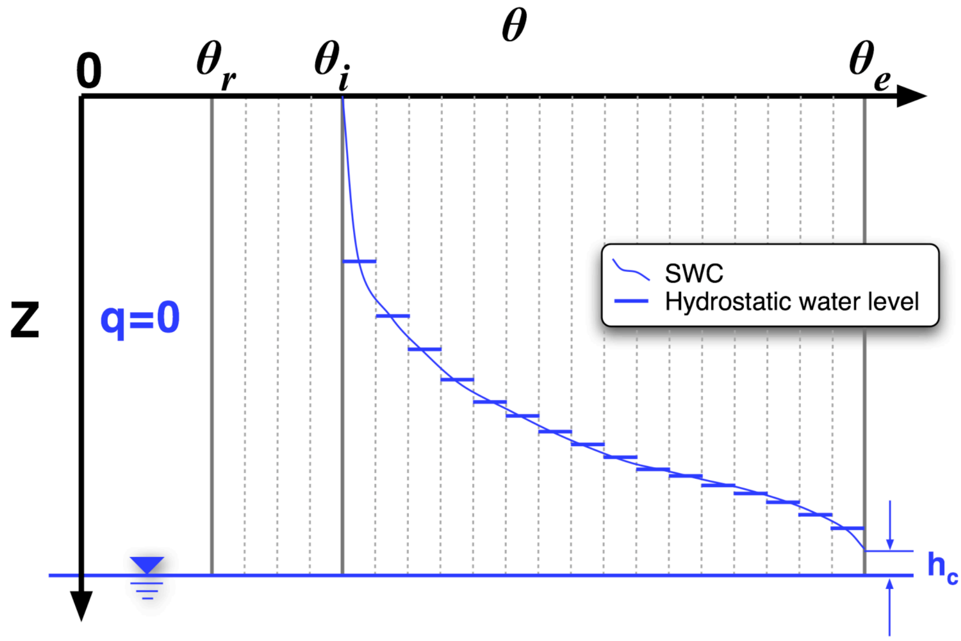

Fig. 2 *θ-Z* bins with SWC.

As the simulation runs, flux boundary conditions at the surface will either deliver water to or remove water from the bins, during precipitation and ET periods, respectively. In the case of precipitation, the volume to be infiltrated will be computed from the flux and time step specifications. Water is first introduced to the left-most bin. If the bin is fully saturated from land surface to D, the layer depth, the amount of water that bin can accept is determined by the product of Ks_i and dt, the time step size.

If the bin is not fully saturated, the wetting fronts in that bin are first advanced according to dZ_i/dt and water sufficient to satisfy demand created by the advancing front is supplied from the amount to be infiltrated. Since dZ_i/dt is dependant on Zi, the length of the saturated portion of the bin, if the bin is unsaturated at the surface, this value is initially undefined. However, when the model is first initialized, "dry bin" wetting front advances are determined via simple trial-and-error and stored for later use as needed. Whether the bin was fully saturated or not, the amount of water infiltrated into the bin is subtracted from the quantity to be infiltrated.

After treating the first bin in this manner, if there is yet water to be infiltrated, the same procedure is followed for each successive bin until either the volume of water to be infiltrated is exhausted or the last bin is reached (Fig. 3). If the last bin is reached, ponding conditions are occurring and water will either pond or contribute to runoff as specified by the user and relevant boundary conditions.

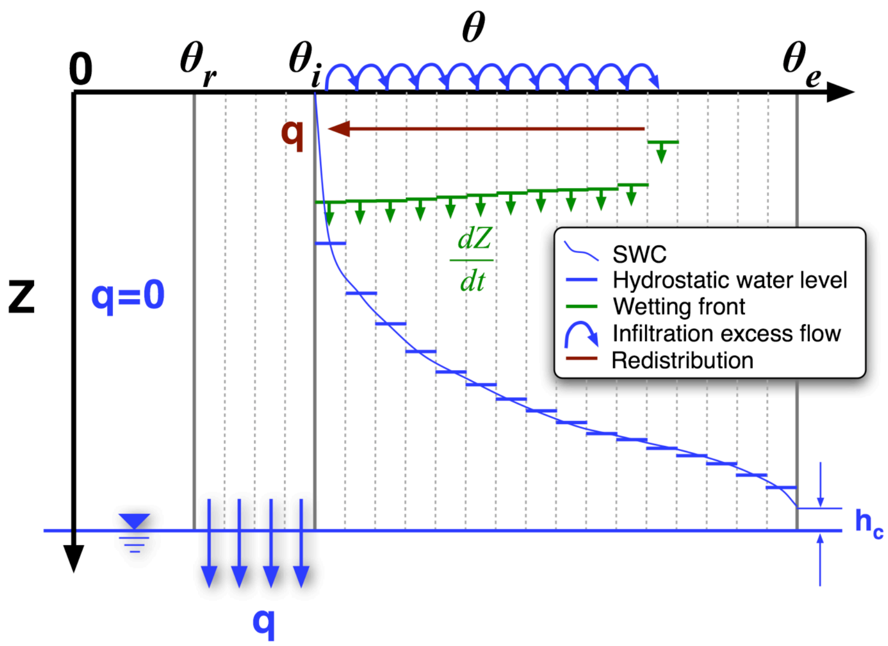

Fig. 3 Model behaviour during infiltration periods.

During ET periods, water is removed from bins with available water above the extinction depth according to their relative suction values. Wetting fronts will continue to advance even though the supply of water from the surface has ceased. Water to supply the deficits in any given bin created by the wetting front advance will be supplied by inter-bin flow from larger-pore bins. Since all bins are interconnected, if there is water available at a given depth in a bin and a smaller-pore bin has a deficit at that depth, water from the largest-pore bin will be used to satisfy the demand. At the end of each time step, a check is also made to redistribute any water as needed based on capillary demands. Since all bins are in contact with one another, if a front in larger-pore bin has advanced beyond the depth of smaller-pore bin fronts, water will be redistributed to the smaller-pore bins according to their G_i values (Fig. 4).

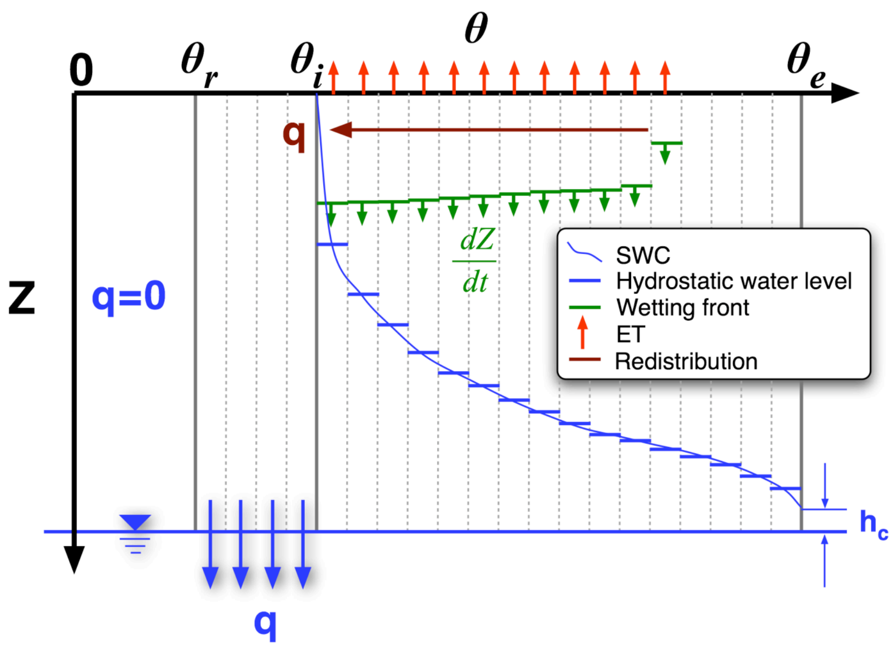

Fig. 4 Model behaviour during non-precipitation periods.

Comparison of Method With RE

The described approach has been developed into a code that simulates one-dimensional (1D) infiltration in, for the time being, a single layer medium. The code is compared here with results from a 1D RE solution obtained from Hydrus1D (Simunek *et al.*, 2005) run under identical soil profile and initial moisture conditions.

The comparison simulations are made using a single-layer system with a initial uniform moisture content and subjected to a constant rainfall rate. The simulated soil is a silt loam with parameters of a saturated hydraulic conductivity of 0.68 cm/hr, porosity, initial and residual moisture contents of 0.501, 0.10, and 0.015, respectively. The Brooks and Corey SWC option was used in both models with the following parameters: a pore size distribution index of 0.234, a bubbling pressure of 20.76 cm and coefficients *a* and *b* equal to 2 and 3, respectively. The depth to the water table was set to 10 cm. The simulation was run for 1.0 hour using a time step size of 30 seconds. Comparison results at t=15 min and t=60 min are shown in Fig. 5 for the bin model approach and in Fig. 6 for the Hydrus1D results.

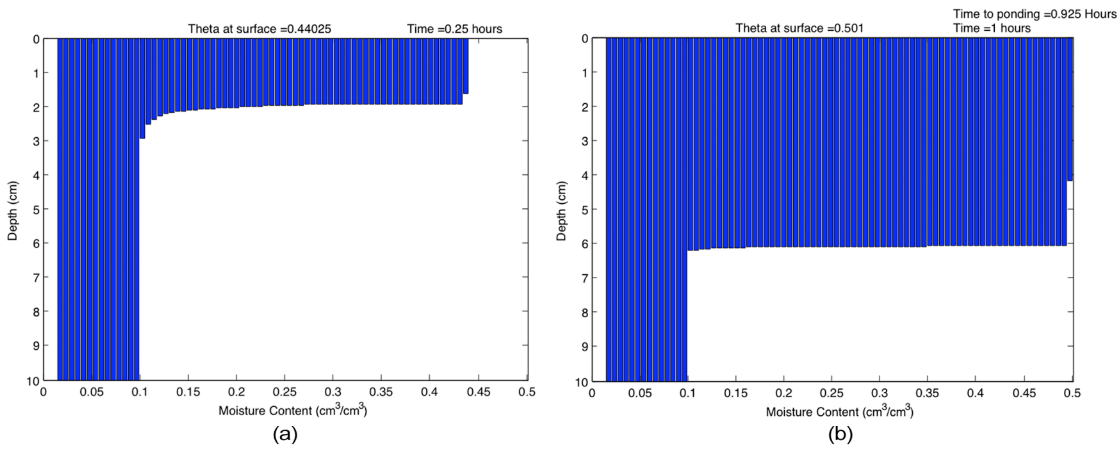

Fig. 5 Bin model solution after **(a)** 15 min. and **(b)** 1 hour.

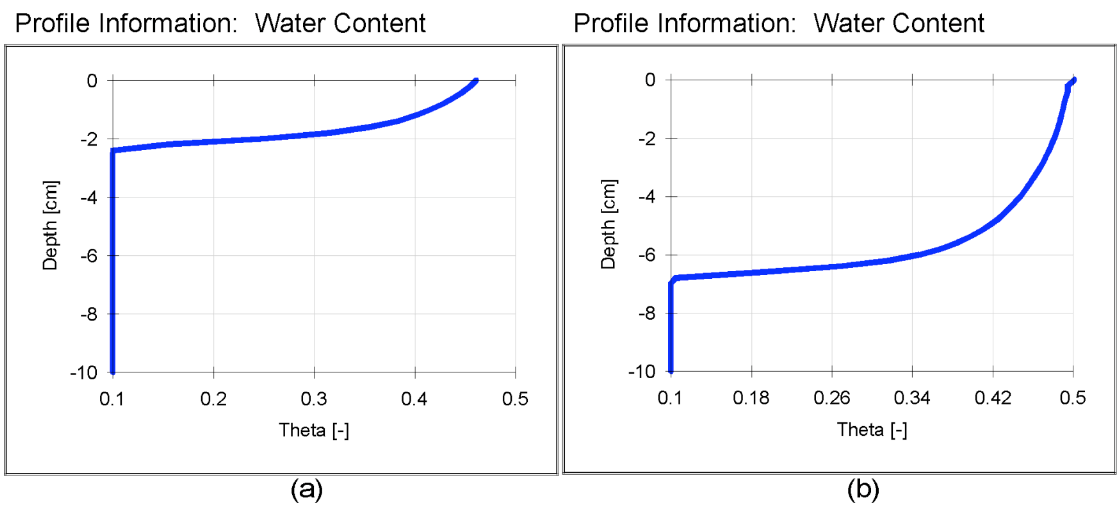

Fig. 6 Hydrus1D solution after **(a)** 15 min. and **(b)** 1 hour.

Comparison of the results indicates good agreement in the shape and depth of penetration of the wetting front as it infiltrates into the soil at both time steps. The Hydrus1D results have more curvature in the wetting front than the bin method throughout the simulation at the large-pore end of the curve. Mass balance checks ensure that the areas under the two wetting front curves are equal. The difference is accounted for by a slightly deeper penetration of the wetting front for the Hydrus1D solutions. The time to ponding calculated by each method was 0.925 hours for the bin method and 0.90 hours for the Hydrus1D solution. It should be noted that the Hydrus1D solution utilized a node spacing of 0.2 cm and takes approximately 3 minutes to run while the bin method takes about 14 seconds and computes its solution using 80 bins in θ-space with effectively one computation node in Z-space.

DISCUSSION AND CONCLUSIONS

The discretized moisture content method offers several advantages to traditional methods of infiltration simulation. Numerical efficiency is achieved by not seeking to estimate the highly non-linear gradients $\partial\psi/\partial Z$ and $\partial\theta/\partial\psi$ mathematically but instead by inferring their value from the simulation of the linear movement of discrete wetting fronts in θ-Z space. In this manner, the bin infiltration method avoids computationally expensive non-linear calculations that often require fine vertical grid resolution and complex solution techniques to achieve numerical stability. Additionally, hydrologically challenging scenarios such as high and rising water tables can be dealt with in the same fashion as deep and well-drained conditions using this method. The method can also be extended to include more complex subsurface conditions such as layered systems with highly varied soil textures. Finally, the redistribution process can be modified to account for coarse grain and macro-pore infiltration conditions where capillary-driven models are incapable of reproducing the unstable wetting fronts that develop in these soils.

The method presented here offers great promise for improving the computational efficiency of coupled surface water-groundwater interaction models as well as any code that simulates the infiltration of water into subsurface soil. Additionally, it can potentially provide the ability to accurately simulate infiltration in coarse grain and macro-pore soils, conditions that are not well supported by current infiltration approaches. In present form, this method deals only with 1D infiltration and evapo-transpiration. However, the discretized moisture content construct upon which this method is based is valid in multi-dimensional conditions and thus could be extended to simulate 3D unsaturated zone flow in future development.

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