A Discretized Moisture Content Infiltration Model

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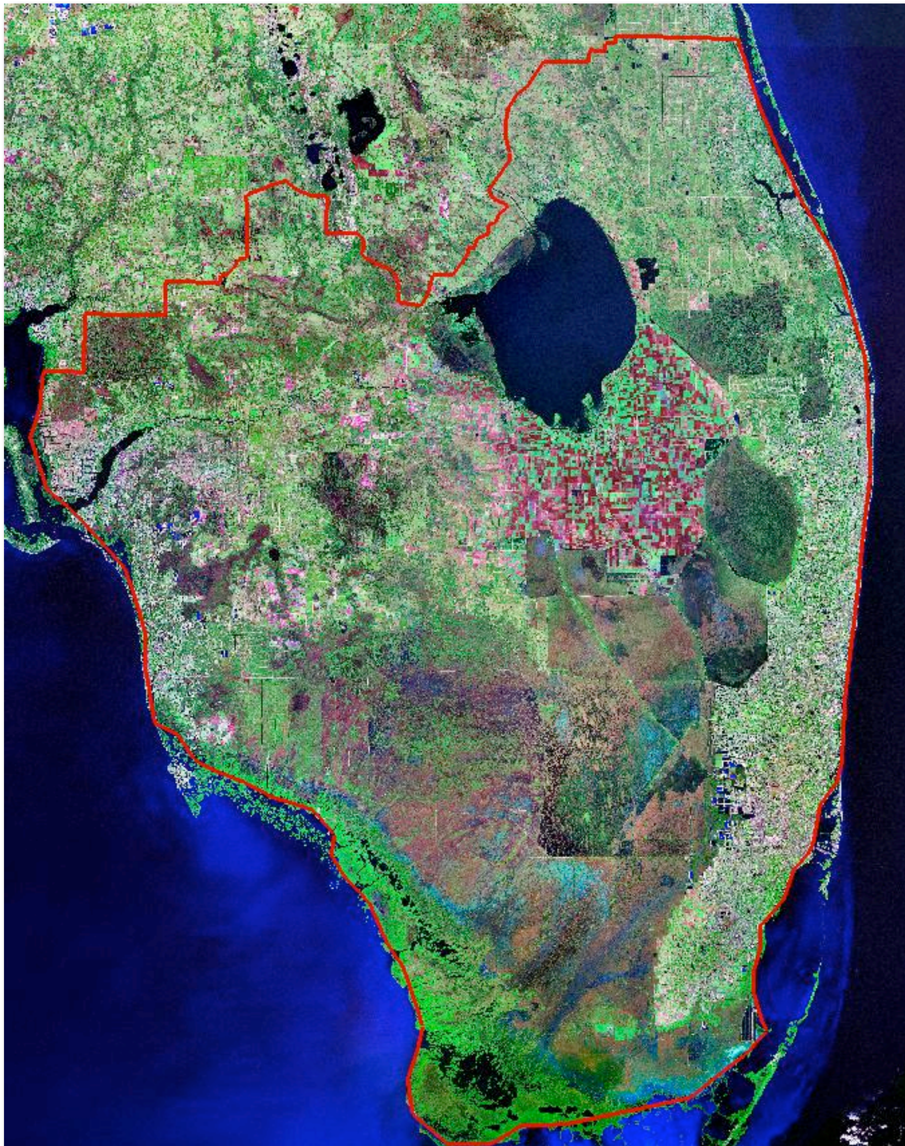
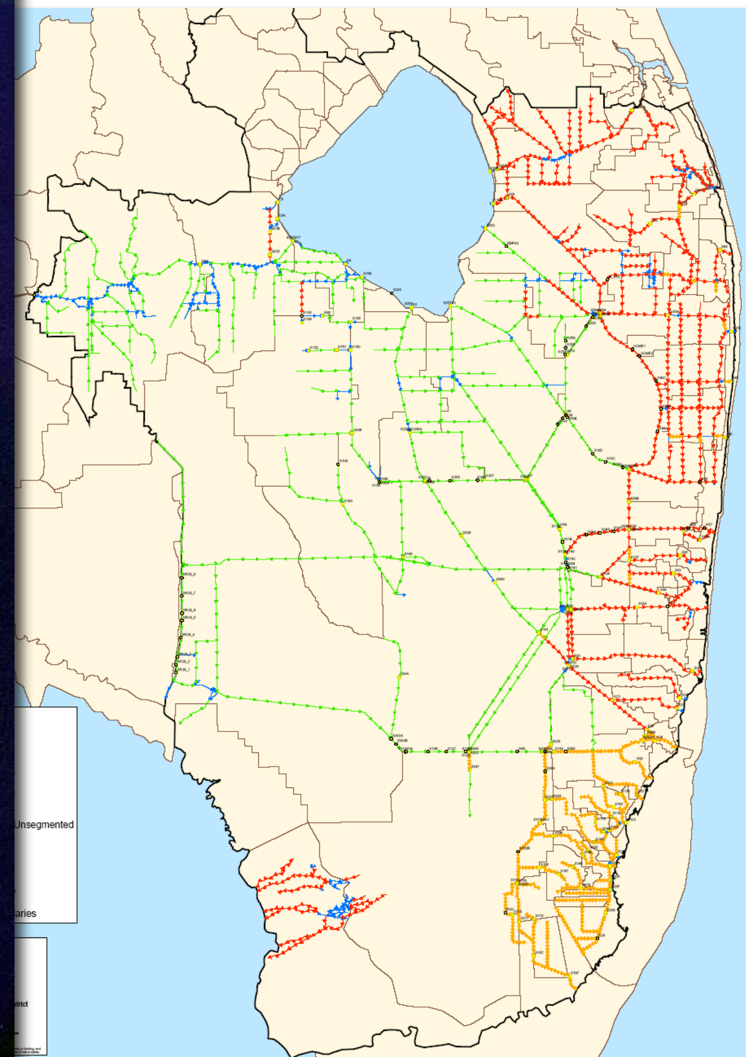
USACE Groundwater-Surface Water Interaction Studies

- Coastal Louisiana
- South Florida/Everglades Restoration

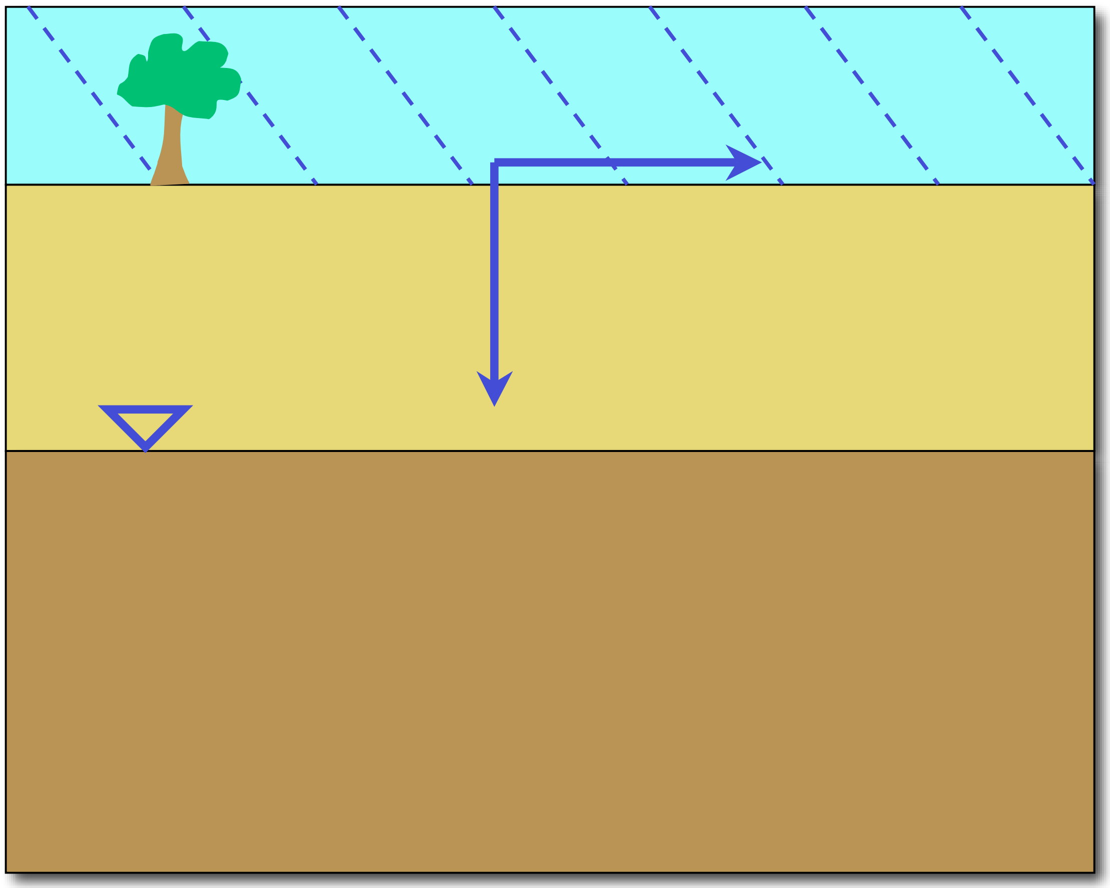
South Florida Challenges





Large-scale GW-SW Interaction Infiltration Modeling Objective

- Get the fluxes correct!



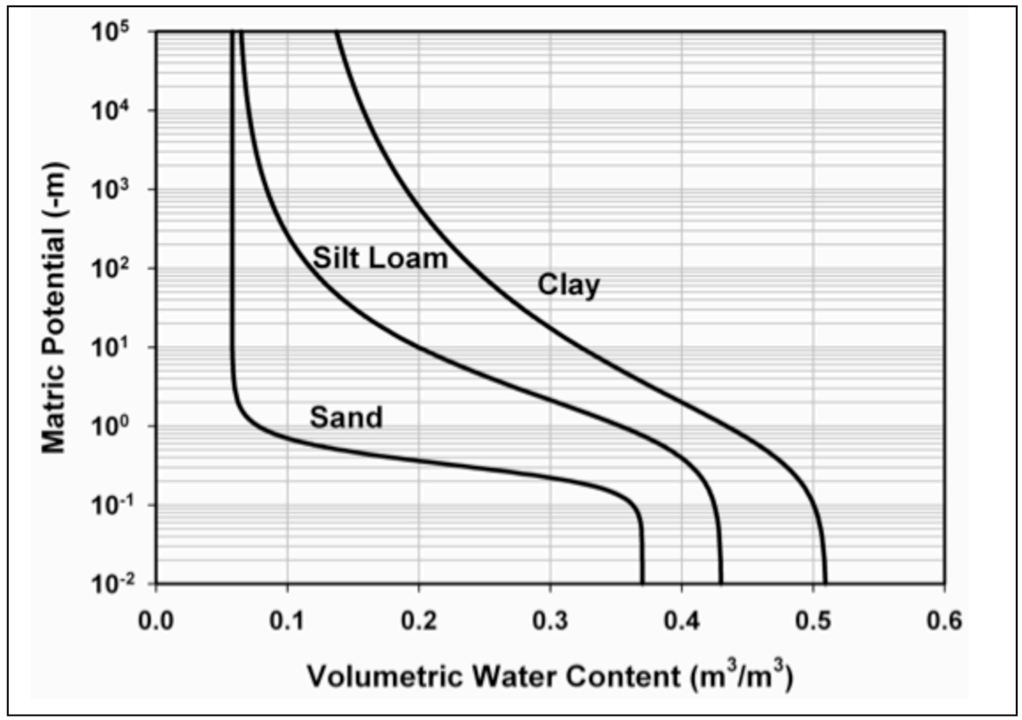
Richards' Equation (1931)

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial Z}\left(K(\theta)\left(\frac{\partial\psi}{\partial Z}+1\right)\right)$$

The 1D mixed form of the Richards Equation where:

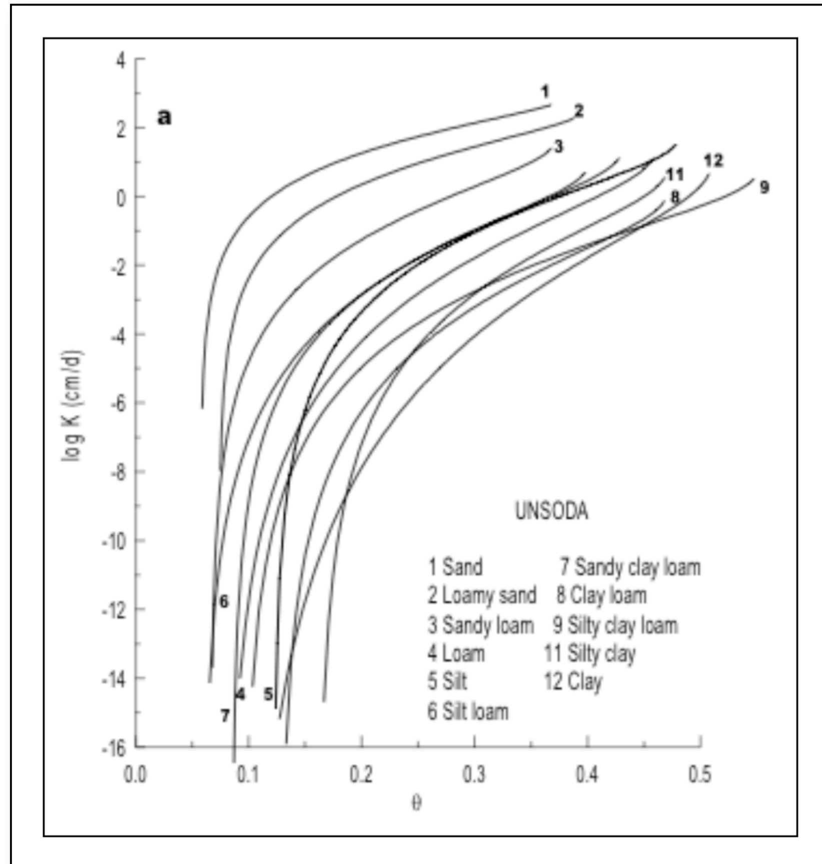
- θ is volumetric soil moisture content (L^3/L^3)
- K(θ) is hydraulic conductivity as a function of θ (L/T)
- ψ is the soil water matric potential (L)
- Z is depth (L)
- t is time (T)

- RE solutions yield θ-ψ or pressure-saturation curves over the entire vadose zone
- Potentially very robust



Vadose Zone Challenges

- RE is highly non-linear due to $K(\theta)$ relationship
- Can commonly vary over 8+ orders of magnitude for a single soil sample



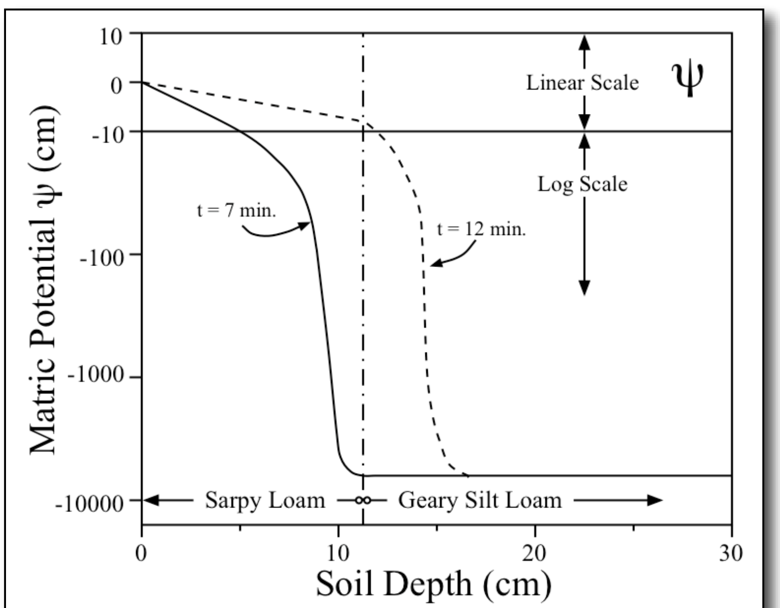
(Leij et al., 1999)

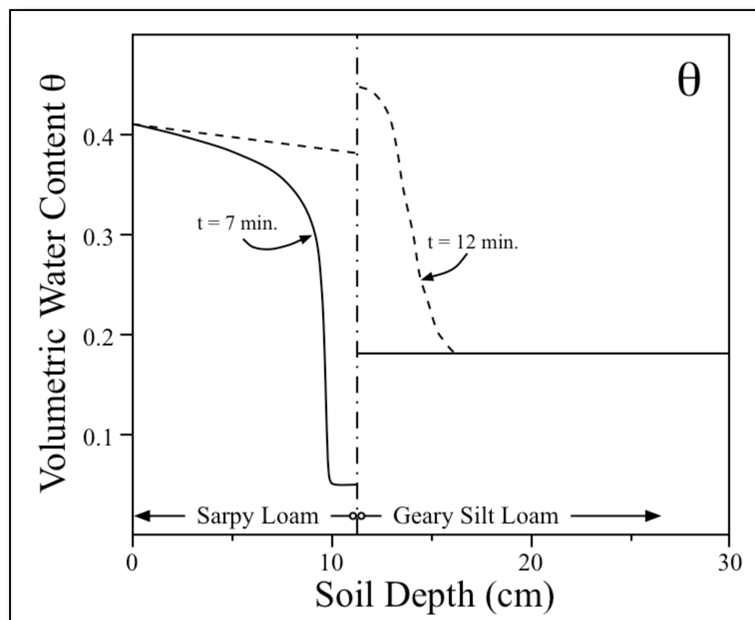
Diffusivity Form of RE

- Define "soil water diffusivity" term

$$D(\theta) = K(\theta)\frac{\partial \psi}{d\theta} \longrightarrow \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left(D(\theta)\left(\frac{\partial \theta}{\partial Z} + K(\theta)\right)\right)$$

- In a layered soil, pressure is continuous, but moisture content is not





Appropriate Vertical Discretization for RE

- Downer & Ogden (2004) addressed the appropriate vertical discretization issue on Hortonian and non-Hortonian watersheds
- Performed spatial (grid) convergence studies using GSSHA with 1D RE to simulate vadose zone fluxes
- Vertical discretization varied from 0.1 to 100 cm (Hortonian) and 0.2 to 50 cm (non-Hortonian) in top 1 m of vadose zone

Downer, CW and FL Ogden. 2004. Appropriate vertical discretization of Richards' equation for two-dimensional watershed-scale modelling. Hydrol. Process. ***18****:1-22.*

Study Results

- Vertical resolution on order of 1 cm was needed near soil surface (but not throughout vadose zone) to properly simulate soil fluxes
- Inadequate resolution can lead to several problems:
  - Large (~2000%!) errors in surface fluxes
  - Erroneous conclusions about the sensitivity of physical parameters in the model
  - Physically unrealistic parameter values
- With proper meteorological inputs and sufficient resolution, RE is able to correct for errors in initial moisture content
- Sufficient resolution at soil surface is critical in proper initiation of infiltration process

Computational Cost of RE

- RE can be computationally expensive when used to describe sharp wetting fronts during infiltration [*Ross*, 1990; *Pan and Wierenga*, 1995]
- RE considered too computationally expensive for use in the context of general surface water hydrology [*Ross*, 1990; *Smith et al.*, 1993; *Short et al.*, 1995; *Corrandini et al.*, 1997]
- Point equation vs. areal-averaged equation

Infiltration Approximation Alternatives

- Dozens of models available
  - Physically-based
    - Philip, Green & Ampt, Smith-Parlange…
  - Semi-empirical
    - Horton, Holtan, Singh-Yu, Overton…
  - Empirical
    - Kostiakov, Huggins-Monke, Collis-George…
- All are limited by simplifying assumptions
  - Deep water table, single layer soil, uniform initial soil moisture, limited boundary conditions, etc…
- None are well suited to accurately simulate gravity-driven, unstable infiltration

Mishra, S.K., J.V. Tyagi and V.P. Singh, 2003. Comparison of infiltration models. Hydrological Processes. 17:2629-2652.

Hypothesis

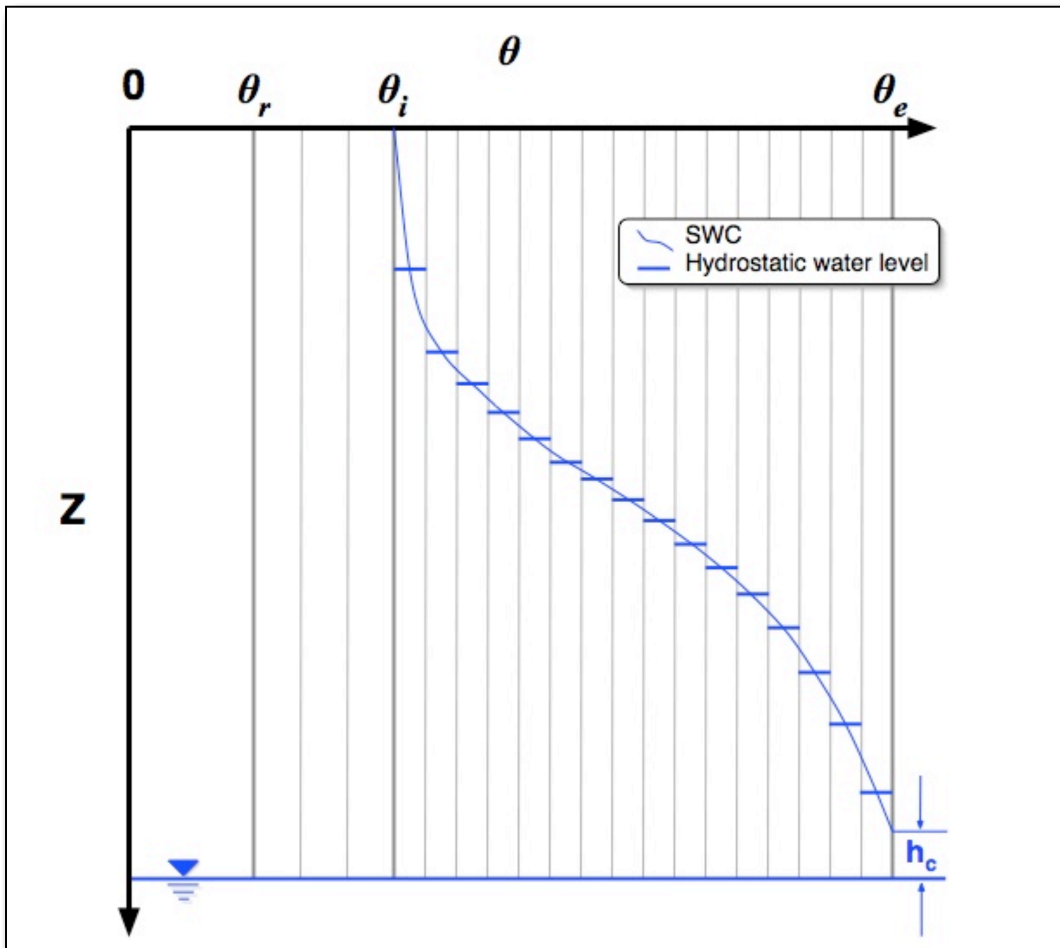
- There exists a more general method to simulate infiltration and the large pore-to-small pore movement of water that occurs in the vadose zone than that offered by RE, it's variants or the other collected approximate solutions.
- Such a method would be capable of effectively simulating capillary- and gravity-driven flow, deal with heterogeneous, layered systems, and a broad variety of initial and boundary conditions

Method Constraints

- Computationally efficient
- Parsimonious
- Robust

Discretized Moisture Content Infiltration Method

- Discretize 1D moisture content-depth space into "bins"



Moisture Content Simulated By Water In Bins

- Bins can only be either saturated or dry
- Saturation zones are tracked by means of “fronts”
- Fronts move under influence of capillarity and gravity:

$$\frac{dZ_i}{dt} = \alpha\left(\frac{K(\theta)_i G_j}{Z_i} + K(\theta)_i\right)$$

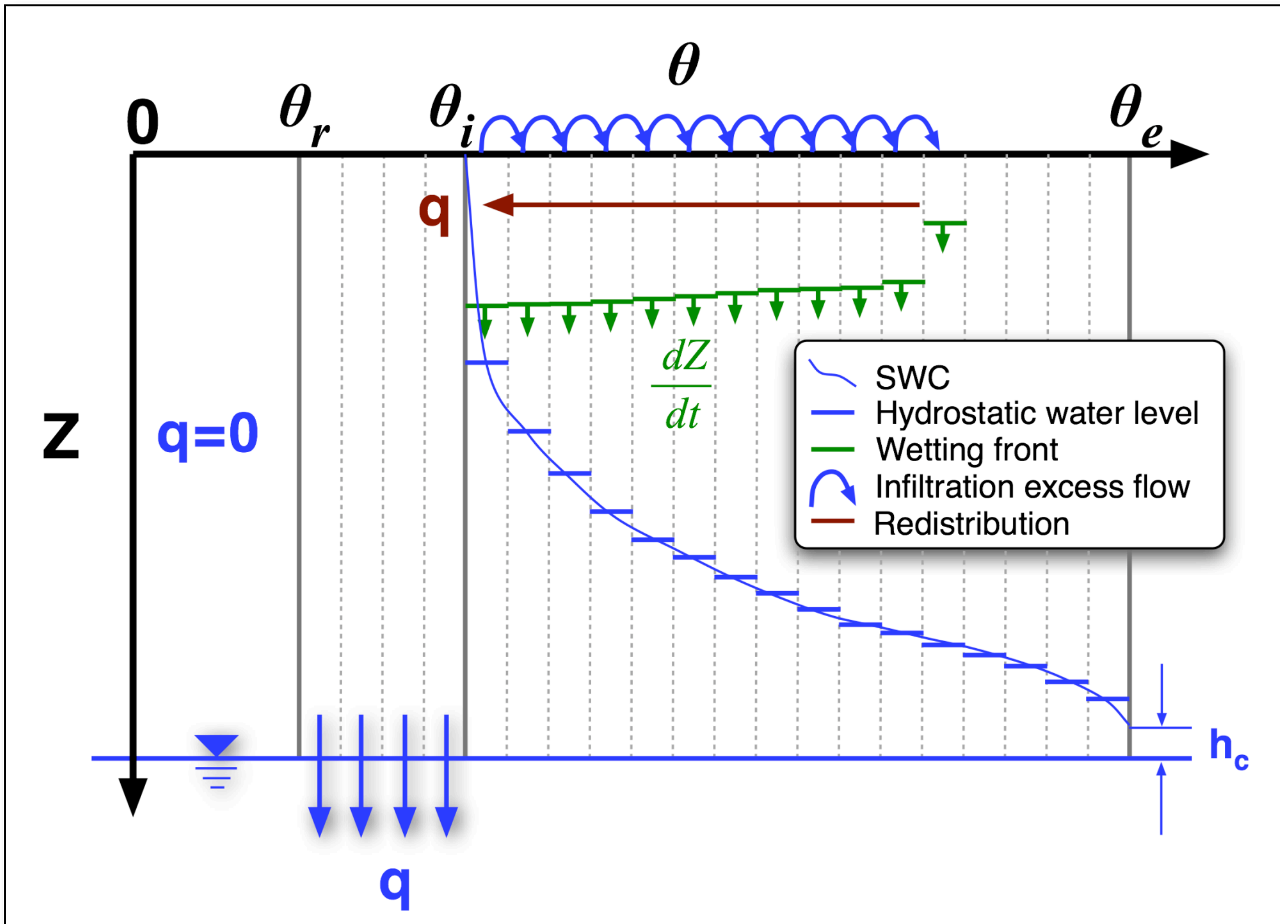
- Hydraulic conductivity function (Jackson, 1972):

$$K(\theta_j) = K_s \left(\frac{\theta_j + \frac{\Delta\theta}{2}}{\theta_s}\right)\frac{\sum_{j=i}^{M}\frac{2j+1-2i}{\psi_j^2}}{\sum_{j=1}^{M}\frac{2j-1}{\psi_j^2}}$$

- G_j values are the suction head for the wetting front taken from pressure saturation curve

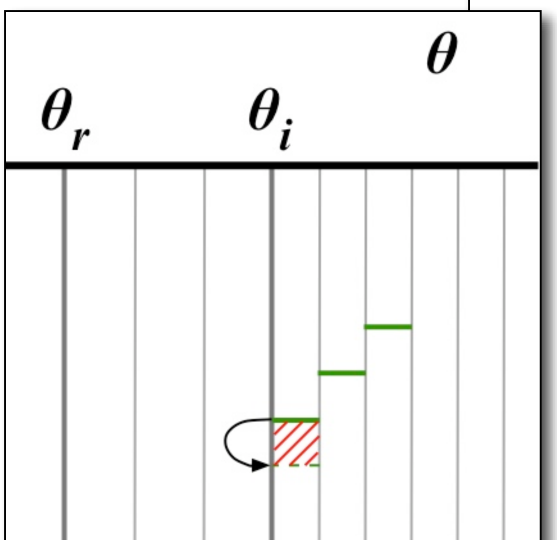
Infiltration

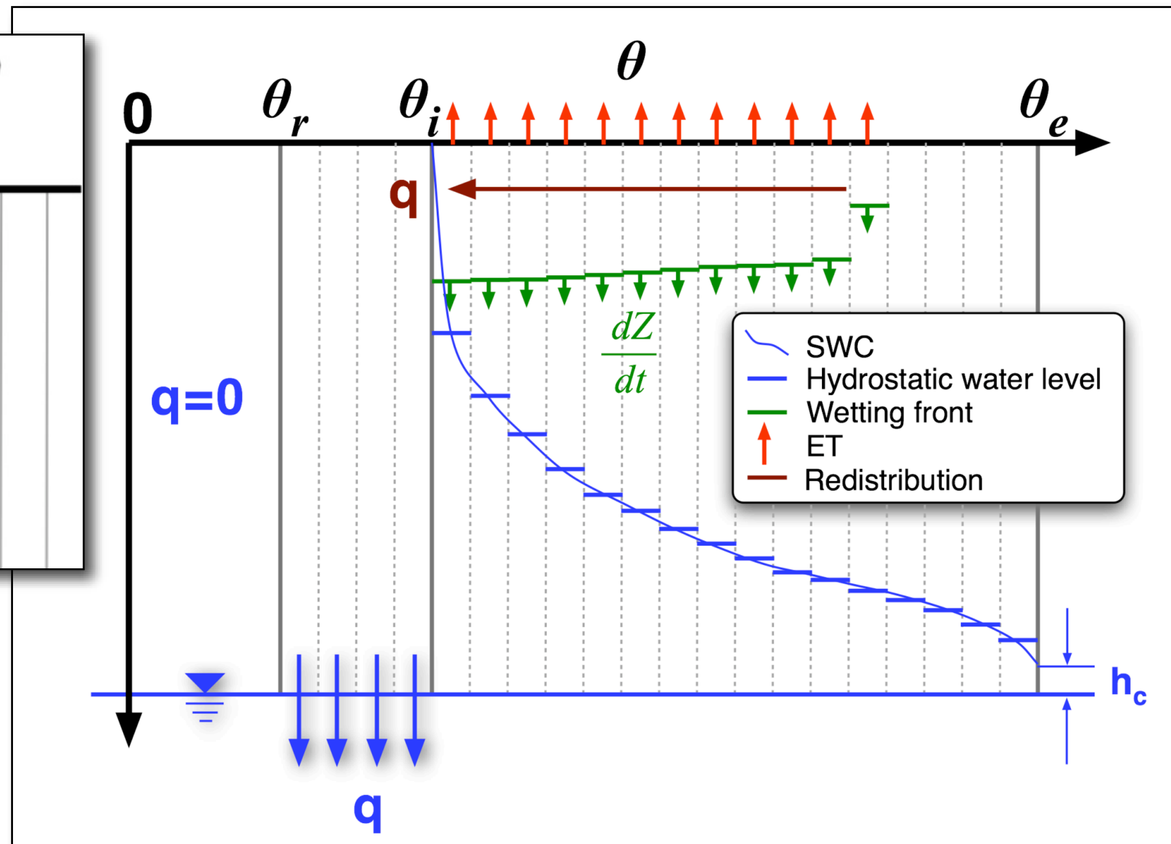
- Boundary fluxes are distributed over bins according to bin front speed



Non-Precipitation Periods

- Fronts advance creating soil moisture deficits which induces flow from right bins
- ET is removed





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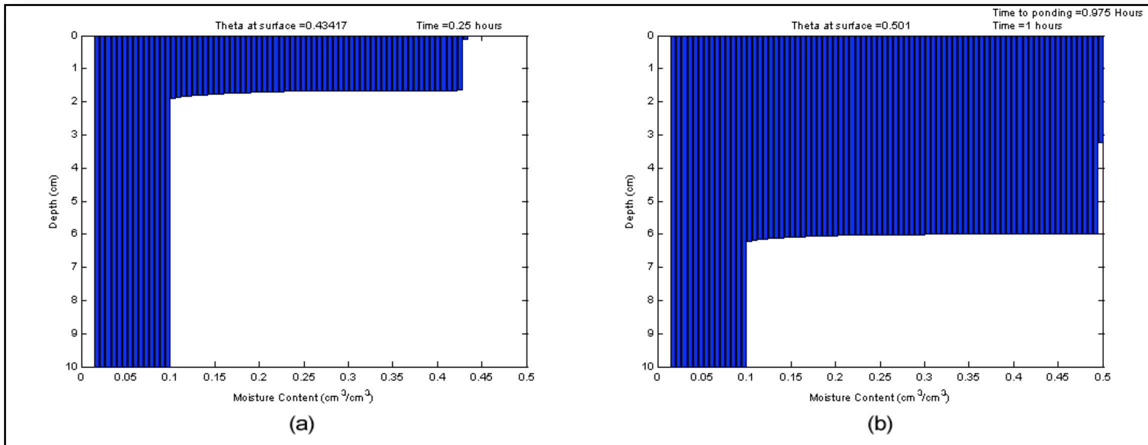
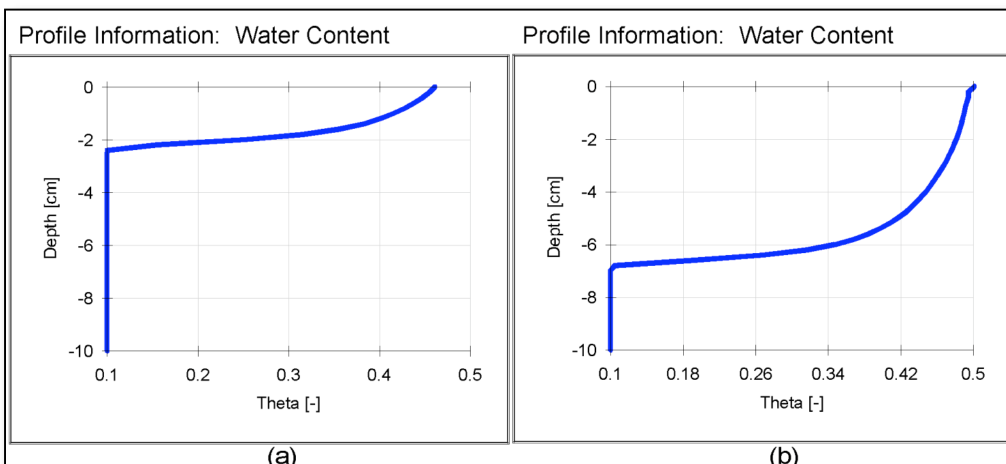
Comparison with RE

- Silt Loam (K_s = 0.65 cm/hr, θ_r = 0.015, θ_i = 0.10, θ_e = 0.501), 2.5 cm/hr rain for 1.0 hours

Discretized MC



1-D RE (Hydrus1D)



Comparison with RE

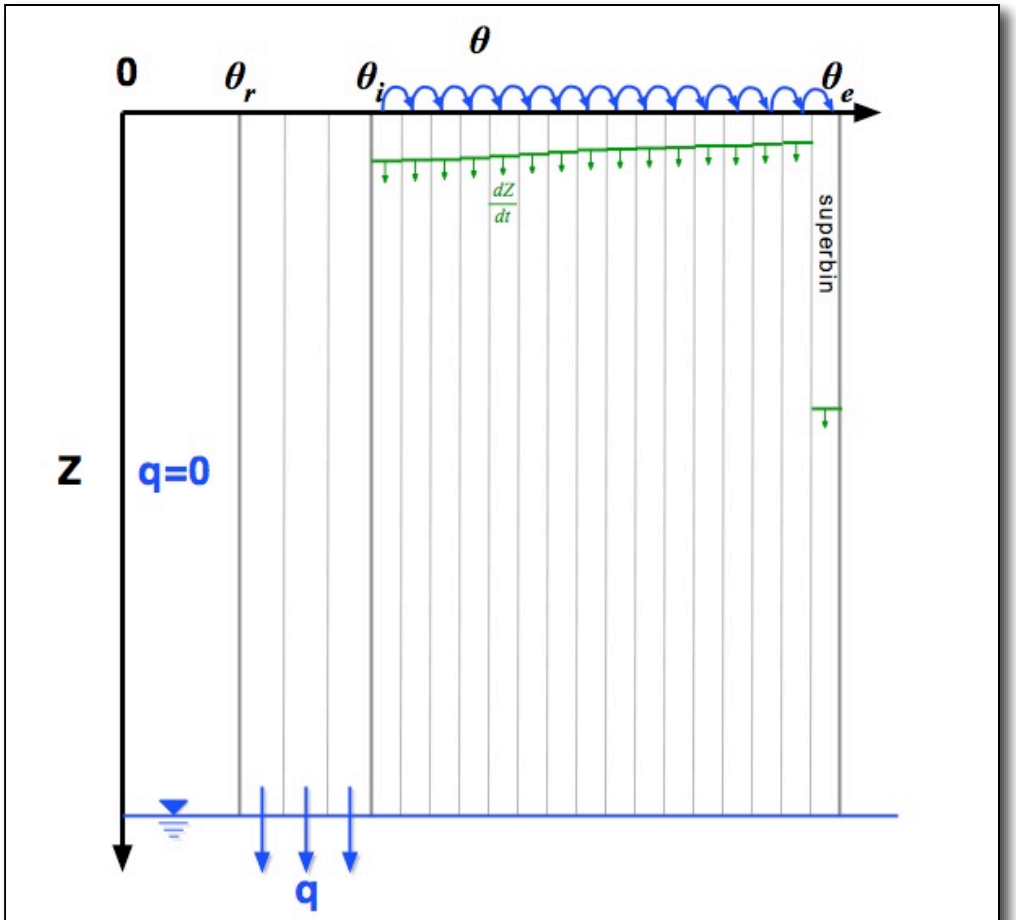
- Time to ponding: 0.967 hrs (DMC), 0.90 hrs (RE)
- Wetting front depth: 6.2 cm (DMC), 6.7 cm (RE)
- Cumulative Infiltration: 2.5 cm (DMC), 2.5 cm (RE)
- Computation time: 14 seconds (DMC), 180 seconds (RE)
- Curvature of DMC method at large-pore end is more piston-like than RE

Method Advantages

- Method can produce infiltration estimates that compare favorably with RE
- Numerical efficiency gained because method does not require direct estimation of $\partial K/\partial Z$, $\partial \theta/\partial Z$, $\partial \psi/\partial Z$, or $\partial \phi/\partial Z$
- Simulation of layered soils and preferential flow is possible
  - One computation cell per layer type
  - Internal ponding at layer boundaries prior to imbibition into that layer

Preferential Flow Simulation

- Preferential flow simulated with "superbins"



Current Development Focus

- Proper form of α in dZ/dt equation

$$\frac{dZ_i}{dt} = \alpha\left(\frac{Ks_i G_i}{Z_i} + Ks_i\right)$$

currently using: $\alpha = \dfrac{2}{\theta_e - \theta_i}$

- Apply method to multi-layered systems
- Apply method to macro-pore/preferential flow systems

Questions or Comments?

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