A Toolbox of Infiltration Models for Surface Water-Groundwater Interaction

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Introduction

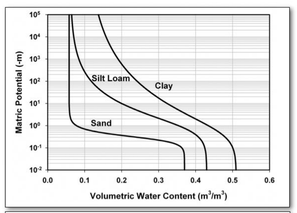
The fundamental role of infiltration in surface and subsurface hydrology has attracted a great deal of attention from researchers for over a century and lead to the development of numerous models for its computation. The most general of these is the Richards' equation (RE), which can be used to compute soil moistures and hydrological fluxes over a wide range of field conditions such as soil layering, shallow groundwater tables and both infiltration and saturation excess runoff mechanisms in a single watershed. However, applying RE in hydrological models is often computationally very expensive due to the highly non-linear nature of the unsaturated hydraulic conductivity, K(θ). Being a soil moisture content, or θ-based, or capillary-driven model, RE is also limited in its applicability by physical conditions that produce unstable, gravity-driven infiltration behavior. Thus, when considering coupled surface and subsurface flow systems, physical and computational factors motivate the search for alternatives to RE in simulating the quantity and rate of infiltration. Research is underway to identify a toolbox of infiltration models that are selected for coupled surface water-groundwater interaction applications based on physical and practical considerations.

Background

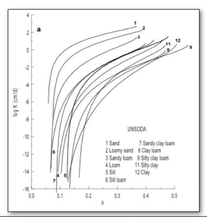
$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left(K(\theta)\left(\frac{\partial \psi}{\partial Z}+1\right)\right)$$

The 1D mixed form of the Richards Equation where:
- θ is volumetric soil moisture content (L^3/L^3)
- K(θ) is hydraulic conductivity as a function of θ (L/T)
- ψ is the soil water matric potential (L)
- Z is depth (L)
- t is time (T)

Solving RE for ψ over the possible range of θ yields a non-linear soil water characteristic (SWC) curve. The non-linear nature of this curve is due to the K(θ) function. Models that apply RE to simulate infiltration define an SWC for each soil simulated in the model.



Soil Water Characteristic (SWC) curves for various soil textures.



Unsaturated hydraulic conductivity relationships for UNSODA database (after Leij et al., 1999).

Due to the highly non-linear nature of K(θ), which can commonly span over 8 orders of magnitude in natural soils (Leij et al., 1999), there is often a large computational burden when using RE to model infiltration, particularly in describing sharp wetting fronts (Ross, 1990; Pan and Wierenga, 1995). Due to the computational burden, modelers are tempted to use reduce the number of nodes in the unsaturated zone of their models. However, this often leads to increased numerical error. Downer and Ogden (2003) and van Dam and Feddes (2000) found that vertical resolutions on the order of 1 cm are typically needed for surface soil water fluxes to be accurate. This level of discretization can dramatically increase the computational burden of the simulations.

Often, simpler, physically based, semi-empirical and empirical models can be appropriately used, particularly when there is considerable uncertainty in defining K(θ) in a given soil. Mishra, Tyagi and Singh (2003) comparatively evaluated 14 popular, representative infiltration models on 243 sets of infiltration data collected from field and laboratory tests conducted in India and the USA on 23 different soils ranging from coarse sand to fine clay. Their comparison revealed that models from each category (empirical, semi-empirical and physically based) performed well in simulating the infiltration behavior of most tested soils.

Physically-based Models	Semi-empirical Models	Empirical Models
Philip	Singh-Yu	Kostiakov
Green-Ampt	Mishra-Singh	Modified Kostiakov
Linear Smith-Parlange	Horton	Huggins-Monke
Non-linear Smith-Parlange	Holtan	Collis-George
Smith	Overton	

However, their study also demonstrated that for the infiltration data from 7 coarse-grained soils, none of the models was able to satisfactorily reproduce the observed infiltration behavior. For another 4 coarse-grained soils, a small handful of models performed only marginally better. The authors concluded that "the infiltration data sets of [these] soils generally showed an erratic behavior of the infiltration decay pattern, which cannot be simulated using [the tested] simple models." RE and the simpler approximations used in this study are capillary-driven models, and as such are unable to accurately simulate unstable flow.

Predicting Unstable Infiltration

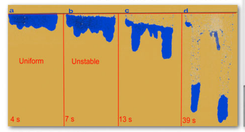
Fluid migration in porous media is a result of the interplay between gravitational, capillary, pressure and viscous forces (Su *et al.* 1999). The Bond number (B_o) is the ratio of gravitational to capillary forces as defined by:

$$B_o = \frac{(\rho_f - \rho_a) g a^2}{\sigma_{f-a}}$$

where ρ_f and ρ_a are the fluid and air densities, respectively, g is the gravitational acceleration, a is the typical pore size, and σ_{f-a} is the surface tension of fluid in air. Prazak *et al.* (1992), Su *et al.* (1999) and others have shown that fluid flow instability in soils occurs when gravitational forces approach ~5% of capillary forces ($B_o \approx 0.05$). Or and Ghezzehei (2003) suggest this behavior can begin at values as low as 1% ($B_o \approx 0.01$).

Stable infiltration

Unstable infiltration



(Nicholl *et al.*, 1994)

Research indicates that B_o can be used as a diagnostic to predict unstable infiltration behavior and thus the conditions where RE and other capillary-based models are not appropriate for simulating infiltration in a soil.

A Toolbox of Infiltration Models for Coupled Surface Water-Groundwater Interaction Applications

As part of the US Army Corps of Engineers (USACE) seven-year System-Wide Water Resources Program (SWWRP), research is underway to achieve significantly faster model run times for surface water-groundwater (SW-GW) interaction applications by means of adjusting the number and complexity of physical processes applied to a particular problem, as warranted by the study needs and physical parameters. Currently most SW-GW models used within the USACE employ various forms of RE to calculate fluid fluxes between the surface water and saturated subsurface domains. The computational cost of the wide application of RE in these models limits the use of SW-GW interaction codes in problems where such tools are needed to accurately simulate coupled systems.

By seeking connections between vadose zone flow formulations and physical characteristics that affect infiltration such as soil types, topography, initial soil moisture and diagnostics such as B_o, a protocol is being developed that will guide the selection of physically appropriate infiltration formulations for the conditions and study needs.

MINC Infiltration Model

In addition to the protocol, another effort to improve SW-GW model run times by use of alternatives to RE is the development of a multiple interactive continua (MINC) infiltration model. MINC models are often used in fractured flow modeling where the highly disparate flow regimes of the rock matrix and fracture network must both be considered when attempting to simulate the movement of fluid through the media (Wu et al., 2004). In this MINC infiltration model, θ-Z space is discretized into bins wherein the movements of hydrostatic, wetting and drying fronts are computed over time.

The multiple continua of each θ-bin interact by the exchange of water from large-θ to small-θ bins as driven by capillarity (see figures on next panel). During precipitation events, if a bin becomes saturated at the surface, excess water is sent to successively larger bins until the soil is completely saturated and runoff occurs. During non-precipitation times, evapo-transpiration (ET) removes water from bins and redistribution occurs as dictated by capillarity. Water that has entered the bins is represented by a wetting front that migrates at a speed dictated by the following unsaturated form of Darcy's law (Smith et al. 1993):

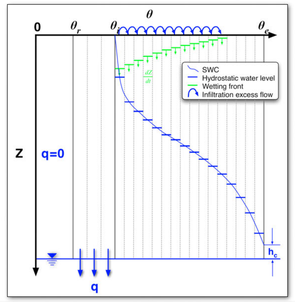
$$\frac{dZ}{dt} = \frac{1}{(\theta_o - \theta_i)}\left(\frac{K_s G(\theta_i, \theta_o)}{Z} + K(\theta_o)\right)$$

where θ_i and θ_o are the moisture contents at the left and right sides of each bin, respectively, K_s is the saturated hydraulic conductivity, and G is the capillary drive term as given by (Ogden and Saghafian, 1997):

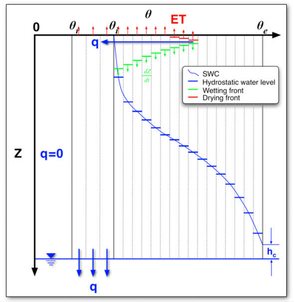
$$G(\theta_i, \theta) = H_c\left(\frac{\Theta^{3+1/\lambda} - \Theta_i^{3+1/\lambda}}{1 - \Theta_i^{3+1/\lambda}}\right)$$

where H_c is the Green & Ampt wetting-front capillary pressure parameter, $\Theta = (\theta - \theta_r)/(\theta_e - \theta_r)$ is the relative saturation, and λ is the soil pore size distribution index.

MINC Infiltration Model (cont.)



MINC infiltration model during precipitation event. Wetting fronts advance at dZ/dt. If a bin is saturated, overflow to next bin occurs until the soil is fully saturated and runoff occurs.



MINC infiltration model during non-precipitation. Wetting and drying fronts advance at dZ/dt. If a saturated bin loses water to flow to the water table or ET, water flows from the next larger bin to satisfy the deficit.

Wetting and drying fronts advance until the volume of water in that bin is exhausted by ET or redistribution, or until the front encounters the hydrostatic front. Antecedent soil moisture conditions set the initial position of the hydrostatic front in each bin based on a van Genuchten or Brooks and Corey approximation.

The position of the water table is not static during the simulation, which allows this approach to model the saturation of the soil column from either beneath or above. In contrast, many of the simpler infiltration models are limited in their ability to handle high and rising water tables. While RE describes a continuous ψ-θ relationship in the vadose zone by generating a continuous SWC for the entire profile, this model simulates an SWC by means of tracking the vertical movement of fronts in and the exchange of water between bins. Finally, this method is explicit and does not benefit from additional grid nodes in the vadose zone, thus reducing the computational burden of high spatial resolution and non-linear solving requirements.

Conclusions

The USACE is conducting research in the development of a protocol to guide the selection of appropriate infiltration models for use in SW-GW interaction studies based on the physical conditions and practical needs of the study. Development of a generalized infiltration model based on MINC principles appears to offer distinct advantages to RE alternatives while also dramatically reducing the computational burden of RE approaches.

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